Renormalizability of θ-expanded chiral electrodynamics

Maja Burić

with D. Latas, V. Radovanović & J. Trampetić

Gravity: new ideas for unsolved problems
Divčibare, 12-14 September 2011

Outline

Noncommutativity of coordinates was introduced to regularize divergences in QFT.

Only a few theories defined on the Moyal space are renormalizable: these include the Grosse-Wulkenhaar ϕ^4 model and also some θ-expanded gauge models. For the latter, renormalizability is checked in θ-linear order and at one loop.

Definition of consistent quantized noncommutative field theories is a very important issue which we have to address and hopefully solve.

Also, we should analyse potentially measurable characteristic effects of noncommutativity and compare them with the experimental data.

The Moyal space is most frequently used as a background noncommutative space because it is flat. It can have both Minkowski and Euclidean signatures.

More importantly, Moyal space has a well defined representation of fields by functions of commutative coordinates x^μ , while the product of functions is the noncommutative Moyal-Weyl $\star$-product: $\star = \exp\left(\frac{i}{2}\,\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}\right)$. Trace is equal to the usual integral.

Commutation relation $[x^\mu \overset{\star}{,} x^\nu] = i\theta^{\mu\nu} = \text{const}$ obviously breaks Lorentz symmetry; it preserves translations.

Moyal space representation gives a well defined calculational scheme for a field theory, for perturbative quantization for example.

Gauge symmetries on the Moyal space can be represented in various ways. If we keep the Lie group-description, noncommutativity restricts possible groups to U_n and representations to the fundamental and adjoint. The obstruction to renormalizability is typically the UV/IR mixing.

If we modify the original symmetry by allowing the gauge potential to take values in the enveloping algebra, we get symmetries in some ways closer to the Standard model, as it becomes possible to represent SU_n groups and the tensor products of groups. Our model is defined in this setup.

Relation between noncommutative and commutative gauge symmetries in the enveloping-algebra or θ-expanded approach is given through the Seiberg-Witten expansion.

We start with two gauge symmetries, the commutative one and its adjoined noncommutative generalization. The SW expansion relates the infinitesimal noncommutative symmetry transformations, which have values in the enveloping algebra, with the corresponding commutative ones. The expansion is in the symmetrized products of the group generators and at the same time in parameter of noncommutativity θ.

SW map also relates matter and gauge fields $\hat{\varphi}$, $\hat{A}_\mu$, $\hat{F}_{\mu\nu}$ with φ, A_μ, $F_{\mu\nu}$. In first order:

$$\hat{\varphi} = \varphi + \frac{1}{2} q\,\theta^{\mu\nu} A_\mu \partial_\nu \varphi + \dots$$

$$\hat{A}_\rho = A_\rho + \frac{1}{4} q\,\theta^{\mu\nu}\{A_\mu, \partial_\nu A_\rho + F_{\nu\rho}\} + \dots$$

$$\hat{F}_{\rho\sigma} = F_{\rho\sigma} - \frac{1}{2} q\,\theta^{\mu\nu}\{F_{\mu\rho}, F_{\nu\sigma}\} + \frac{1}{4} q\,\theta^{\mu\nu}\{A_\mu, (\partial_\nu + D_\nu) F_{\rho\sigma}\} + \dots$$

The SW map given above is not unique. It was shown that a whole class of solutions to the closure equations can be obtained by a shift of fields

$$A_\mu^{(n)} \to A_\mu^{(n)} + \mathrm{A}_\mu^{(n)}, \quad \varphi^{(n)} \to \varphi^{(n)} + \Phi^{(n)},$$

where $\mathrm{A}_\mu^{(n)}$ and $\Phi^{(n)}$ are arbitrary gauge covariant expressions of given dimension and in given order in θ.

This means that, assuming that noncommutative fields are primary or fundamental, we cannot say beyond zeroth approximation which commutative field is a physical one, the one which we observe and measure in the experiments.

Perhaps all SW mappings are equivalent? If not, how to choose one?

The idea of the Vienna group in the 00's was that the SW freedom could be used to identify, among equivalent actions, the one which is renormalizable is based on the fact that SW gives additional counterterms which can be used for renormalizability.

This worked for noncommutative U_1. However, quantization of noncommutative electrodynamics described by the action

$$S = \int \bar{\hat{\psi}} \star \left(\gamma^\mu (i\partial_\mu - q\hat{A}_\mu) - m\right) \star \hat{\psi} - \frac{1}{4} \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}$$

gave divergent term $\theta_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}\gamma_\rho\psi\,\bar{\psi}\gamma_5\gamma_\sigma\psi$ which could not be removed.

This term however vanishes identically for chiral electrodynamics: this was our motive to reexamine the possibility of its renormalization.

Action

We start with the classical action for noncommutative chiral electrodynamics

$$S = \int i\bar{\hat{\varphi}} \star \bar{\sigma}^\mu(\partial_\mu + iq\hat{A}_\mu) \star \hat{\varphi} - \frac{1}{4}\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}$$

and expand it using the simplest SW expansion of the noncommutative fields. In linear order, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \dots$ we get

$$\mathcal{L}_0 = i\bar{\varphi}\bar{\sigma}^\mu(D_\mu\varphi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{1,A} = \frac{1}{2}q\,\theta^{\mu\nu}\Big(F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\Big)$$

$$\mathcal{L}_{1,\varphi} = \frac{i}{16}q\,\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\,F_{\alpha\beta}\,\bar{\varphi}\,\bar{\sigma}^\rho(D_\gamma\varphi) + \mathrm{h.c.}$$

Δ is cyclic and completely antisymmetric, $\Delta^{\alpha\beta\gamma}_{\mu\nu\rho} = -\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{\mu\nu\rho\delta}$.

Next, we quantize perturbatively, by functional integration; we calculate only divergent terms in the effective action and at one loop.

To keep the (commutative) gauge covariance explicit we use the background field method.

To compute the functional integral usually one complexifies the gauge field or introduces the Majorana spinors: we work with the Majorana spinor, ψ. We thus have

$$\mathcal{L}_{0,\psi} = \frac{i}{2}\,\bar{\psi}\gamma^{\mu}(\partial_{\mu} - \mathrm{i}q\gamma_5 A_{\mu})\psi, \;\; \mathcal{L}_{1,\psi} = \frac{i}{16}\,q\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\,F_{\alpha\beta}\bar{\psi}\gamma^{\rho}(\partial_{\gamma} - \mathrm{i}q\gamma_5 A_{\gamma})\psi,$$

which looks as an axial symmetry; more complicated expressions however are not Majorana-covariant.

First we need write the quadratic part of the action in the form

$$S^{(2)} = \int \begin{pmatrix} \mathcal{A}_\kappa & \bar{\Psi} \end{pmatrix} \mathcal{B} \begin{pmatrix} \mathcal{A}_\lambda \\ \Psi \end{pmatrix}.$$

Then we adjust both fields to have the same propagator by multiplying by a constant matrix $\mathcal{C}$

$$\mathcal{B}\mathcal{C} = \Box\mathcal{I} + N_1 + T_1 + T_2.$$

The one-loop effective action is obtained from the perturbation expansion

$$\Gamma^{(1)} = \frac{i}{2}\,\mathrm{STr}\log\left(\mathcal{I} + \Box^{-1}N_1 + \Box^{-1}T_1 + \Box^{-1}T_2\right).$$

The interaction vertices in the previous formula are

$$N_1 = q \begin{pmatrix} 0 & -i\bar\psi\gamma_5\gamma^\lambda\not\partial \\ -\gamma_5\gamma^\kappa\psi & i\gamma_5\not A\not\partial \end{pmatrix}$$

$$T_1 = -q \begin{pmatrix} V^{\kappa\lambda} & -\frac{1}{4}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\delta^\kappa_\alpha(\partial_\beta\bar\psi)\gamma^\rho\partial_\gamma\not\partial \\ -\frac{i}{4}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\delta^\lambda_\alpha\gamma^\rho(\partial_\beta\psi)\partial_\gamma & -\frac{1}{8}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}F_{\alpha\beta}\gamma^\rho\partial_\gamma\not\partial \end{pmatrix}$$

$$T_2 = \frac{q^2}{8}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho} \begin{pmatrix} \delta^\kappa_\alpha\delta^\lambda_\beta(\partial_\gamma\bar\psi\gamma_5\gamma^\rho\psi + \bar\psi\gamma_5\gamma^\rho\psi\partial_\gamma) & i\delta^\kappa_\alpha(2\partial_\beta A_\gamma + F_{\beta\gamma})\bar\psi\gamma_5\gamma^\rho\not\partial \\ \delta^\lambda_\alpha\gamma_5\gamma^\rho\psi(2A_\gamma\partial_\beta - F_{\beta\gamma}) & iF_{\alpha\beta}A_\gamma\gamma_5\gamma^\rho\not\partial \end{pmatrix}$$

with $V^{\kappa\lambda} = -\partial_\sigma V^{\sigma\kappa,\tau\lambda}\partial_\tau$ given by

$$
\begin{aligned}
V^{\sigma\kappa,\tau\lambda} = &\frac{1}{2}(g^{\sigma\tau}g^{\kappa\lambda} - g^{\sigma\lambda}g^{\tau\kappa})\theta^{\alpha\beta}F_{\alpha\beta} \\
&- g^{\kappa\lambda}(\theta^{\xi\sigma}F_\xi{}^\tau + \theta^{\xi\tau}F_\xi{}^\sigma) - g^{\sigma\tau}(\theta^{\xi\kappa}F_\xi{}^\lambda + \theta^{\xi\lambda}F_\xi{}^\kappa) \\
&+ g^{\kappa\tau}(\theta^{\xi\lambda}F_\xi{}^\sigma + \theta^{\xi\sigma}F_\xi{}^\lambda) + g^{\sigma\lambda}(\theta^{\xi\kappa}F_\xi{}^\tau + \theta^{\xi\tau}F_\xi{}^\kappa) \\
&- \theta^{\kappa\lambda}F^{\sigma\tau} + \theta^{\kappa\tau}F^{\sigma\lambda} + \theta^{\sigma\lambda}F^{\kappa\tau} + \theta^{\sigma\kappa}F^{\tau\lambda} + \theta^{\tau\lambda}F^{\sigma\kappa} - \theta^{\sigma\tau}F^{\kappa\lambda}.
\end{aligned}
$$

The 3-vertices are contained in N_1 and T_1, the 4-vertices in T_2.

We calculate divergences by dimensional regularization and at the end rewrite them covariantly.

The divergent part of the one-loop effective action is:

$$\Gamma_2 = \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\rho\sigma\tau}(\partial_\lambda F^{\rho\lambda})(\partial_\nu F^{\sigma\tau})$$

$$- \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}\left(i(D^\rho\bar\varphi)\bar\sigma^\sigma(D^2\varphi) + \mathrm{h.c.}\right),$$

$$\Gamma_3 = -\frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\left(\frac{1}{6}F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - \frac{2}{3}F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}\right)$$

$$- \frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\Big(\frac{5i}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma^\rho(D_\nu\varphi) - \frac{\mathrm{i}}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma_\nu(D^\rho\varphi) - \frac{2i}{3}F_{\mu\nu}\,\bar\varphi\bar\sigma^\rho(D_\rho\varphi)$$

$$+ \frac{4}{3}\varepsilon_{\mu\rho\sigma\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\nu\varphi) + \frac{3}{2}\varepsilon_{\mu\nu\rho\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\sigma\varphi)$$

$$+\frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\tau\varphi) + \mathrm{h.c.}\Big).$$

Some of its parts can be immediately recognized as renormalizable, e.g.

$$\Gamma_2 = \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\rho\sigma\tau}(\partial_\lambda F^{\rho\lambda})(\partial_\nu F^{\sigma\tau})$$

$$-\frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}\left(i(D^\rho\bar\varphi)\bar\sigma^\sigma(D^2\varphi) + \mathrm{h.c.}\right),$$

$$\Gamma_3 = -\frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\left(\frac{1}{6}F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - \frac{2}{3}F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}\right)$$

$$-\frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\Big(\frac{5i}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma^\rho(D_\nu\varphi) - \frac{\mathrm{i}}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma_\nu(D^\rho\varphi) - \frac{2i}{3}F_{\mu\nu}\,\bar\varphi\bar\sigma^\rho(D_\rho\varphi)$$

$$+\frac{4}{3}\varepsilon_{\mu\rho\sigma\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\nu\varphi) + \frac{3}{2}\varepsilon_{\mu\nu\rho\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\sigma\varphi)$$

$$+\frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\tau\varphi) + \mathrm{h.c.}\Big)$$

because it is proportional to $\mathcal{L}_{1,A}$. But also it is easy to see that the term

$$\Gamma_2 = \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\rho\sigma\tau}(\partial_\lambda F^{\rho\lambda})(\partial_\nu F^{\sigma\tau})$$

$$- \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}\left(i(D^\rho\bar\varphi)\bar\sigma^\sigma(D^2\varphi) + \text{h.c.}\right),$$

$$\Gamma_3 = -\frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\left(\frac{1}{6}F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - \frac{2}{3}F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}\right)$$

$$- \frac{q^3}{(4\pi)^2\epsilon}\theta^{\mu\nu}\Big(\frac{5i}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma^\rho(D_\nu\varphi) - \frac{\mathrm{i}}{6}F_{\mu\rho}\,\bar\varphi\bar\sigma_\nu(D^\rho\varphi) - \frac{2i}{3}F_{\mu\nu}\,\bar\varphi\bar\sigma^\rho(D_\rho\varphi)$$

$$+ \frac{4}{3}\varepsilon_{\mu\rho\sigma\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\nu\varphi) + \frac{3}{2}\varepsilon_{\mu\nu\rho\tau}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\sigma\varphi)$$

$$+\frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\,\bar\varphi\bar\sigma^\tau(D_\tau\varphi) + \text{h.c.}\Big)$$

will appear in the Lagrangian if we shift the gauge potential A_ρ by $\theta^{\mu\nu}\varepsilon_{\mu\rho\sigma\tau}\partial_\nu F^{\sigma\tau}$, which is an allowed SW field redefinition.

In general, the first-order redefinition

$$A_\mu \to A_\mu + \mathrm{A}_\mu^{(1)}, \quad \varphi \to \varphi + \Phi^{(1)}$$

induces in the action following additional terms:

$$\Delta S^{(1)} = \int (D_\rho F^{\rho\mu})\mathrm{A}_\mu^{(1)} - q\int \bar\varphi\bar\sigma^\mu \mathrm{A}_\mu^{(1)}\varphi + (\int i\bar\varphi\bar\sigma^\mu (D_\mu\Phi^{(1)}) + \mathrm{h.c.})$$

All divergences which we have obtained are of this type, and thus can be removed by counterterms appearing from the SW field redefinitions.

Redefinitions

More concretely, shifts

$$A_\rho \to A_\rho + \frac{1}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\big(\varepsilon_{\mu\rho\sigma\tau}(D_\nu F^{\sigma\tau}) - \varepsilon_{\mu\nu\rho\tau}(\partial^\sigma F_{\tau\sigma}) - 10\varepsilon_{\mu\nu\sigma\tau}(\partial_\rho F^{\tau\sigma})\big),$$

$$\varphi \to \varphi - \frac{i}{12}\frac{q^2}{(4\pi)^2\epsilon}\theta^{\mu\nu}\big(2\sigma_{\mu\nu}D^2 + 8iq\,F_{\mu\rho}\sigma_{\nu\rho} - 5iq\,\theta^{\mu\nu}F_{\mu\nu} + 10q\,\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\big)\varphi$$

transform the effective action to

$$\Gamma^{(1)} = S_{\rm cl} + \frac{4}{3}\frac{q^2}{(4\pi)^2\epsilon}\,\mathcal{L}_{1,A} + \frac{2q^2}{(4\pi)^2\epsilon}\,\mathcal{L}_{1,\varphi}\,.$$

The remaining divergence can be removed by multiplicative renormalization; in principle, noncommutativity parameter gets renormalized too.

The conclusion is therefore, that the ‘good’ commutative action corresponding to the θ-expansion of noncommutative chiral electrodynamics is of the form

$$\begin{aligned}\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm C} + \kappa_1 \mathcal{L}_{1,A} + \kappa_2 \mathcal{L}_{1,\varphi} \\ + \kappa_3 \theta^{\mu\nu} \varepsilon_\mu{}^{\rho\sigma\tau} F_{\rho\sigma} (D^2 F_{\nu\tau}) + i\kappa_4 \theta^{\mu\nu} \big({\rm i} \bar\varphi \bar\sigma_\rho \sigma_{\mu\nu} (D^\rho D^2 \varphi) + {\rm h.c.} \big) \\ + \theta^{\mu\nu} \bar\varphi \big(\kappa_5 F_{\mu\nu} + \kappa_6 \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} + \kappa_7 F_\mu{}^\rho \sigma_{\nu\rho} \big) \varphi + {\rm h.c.}\end{aligned}$$

Running of all couplings has to be checked explicitly to prove renormalizability rigorously.

In the action which we obtained there is an interesting effect: not only the vertices have changed but also the propagators. This means that the dispersion relations are modified in the theory, that is we have an effective change of geometry. On the other hand, this gives yet another possibility for experimental tests.

However in fact, potentially more interesting dispersion relation for photons does not change: the corresponding term turns out to be a 4-divergence. This can be seen also from the modified equation of motion,

$$\partial^\alpha F_{\alpha\beta} - \kappa_3 \theta^{\mu\nu} \left(2\varepsilon_{\mu\alpha\beta\sigma}\eta_{\rho\nu} + \varepsilon_{\mu\rho\sigma\beta}\eta_{\alpha\nu} - \varepsilon_{\mu\rho\sigma\alpha}\eta_{\beta\nu} \right) \partial^\alpha \Box F^{\rho\sigma} = 0$$

as the second term vanishes identically.

This Lorentz-violation effect could have been observed in the CMB spectrum.

But the dispersion relation for fermions changes: the equation of motion for the free particle is

$$(i\bar{\sigma}^\rho \partial_\rho + \mathrm{i}\kappa_4 \theta^{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} \bar{\sigma}^\sigma \partial^\rho \Box)\varphi = 0.$$

To see really the effect let us assume spatial noncommutativity, $\theta^{0i} = 0$ and denote $k^\mu = (E, 0, 0, p)$, $(\theta^{12})^2 = \theta_\perp^2$. We get two solutions

$$k^2 = 0\,, \quad k^2 = \frac{\sqrt{\frac{1}{\kappa_4^2}(\theta_\perp^2 + \theta_\parallel^2) + \theta_\parallel^4 p^4} - \theta_\parallel^2 p^2}{2(\theta_\perp^2 + \theta_\parallel^2)}$$

One of the fermionic modes acquires mass which is for small noncommutativity parameter very large and birefringent.

This could in principle be observed in the neutrino background radiation or in some other astrophysical effect.

- We did not include in the discussion of renormalizability the chiral anomaly: chiral electrodynamics has a nonvanishing anomaly so it can be used just as a building block for a consistent theory, as in the Standard Model.
- Encouraging results on the inclusion of fermions into gauge models were obtained before in 09 by the Madrid group: they found another class of renormalizable models (GUT-inspired, anomaly safe representations of the simple gauge groups).
- These models are in details different: renormalizability is on-shell but it does not neccessitate SW redefinitions; triple-gauge boson interactions are absent; counterterms do not violate parity.
- Can we include the Higgs?

- What does renormalizability in linear order mean? What happens in the next orders, and with the UV/IR mixing?
- It would seem that this model shows that the SW in linear order is not just a redefinition of quantum fields (as shown for Dirac fermions by Grimstrup and Wulkenhaar), as it singles out one specific Lagrangian as (potentially) renormalizable.
- It would be certainly helpful to do some calculations in the second order to get an idea whether there is some systematics. Or perhaps to show that the renormalizability breaks?
- What happens with the noncommutative gauge symmetry in the quantized theory? In the expanded gauge theory NC symmetry relates different orders in θ. Can the corresponding NC Ward identities be of help to analyze relations between different orders, are they explored fully?