Algebraic Bethe Ansatz for deformed Gaudin model

Nenad Manojlović

Group of Mathematical Physics, University of Lisbon
Department of Mathematics, University of Algarve

Gravity: New ideas for unsolved problems
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Outline

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Quantum Integrable Systems

- In the framework of the quantum inverse scattering method (QISM) integrable systems can be classified by underlying dynamical symmetry algebras.
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Spin systems

Model	Quantum $R(\lambda, \eta)$-matrix	Algebra
XXX	rational	Yangian $\mathcal{Y}(sl(2))$
XXZ	trigonometric	quantum affine algebra $\mathcal{U}_q(\widehat{sl}(2))$
XYZ	elliptic	elliptic quantum group $\mathcal{E}_{\tau,\eta}(sl(2))$

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Gaudin Models

- In this sense, one could say that the Gaudin models are the simplest quantum solvable systems being related to classical r-matrices.
- Gaudin models can be seen as a semi-classical limit of the quantum spin systems

$$R(\lambda;\eta) = I + \eta r(\lambda) + O(\eta^2).$$

- Gaudin Hamiltonians are related to classical r-matrix

$$H^{(a)} = \sum_{b \neq a} r_{ab}(z_a - z_b).$$

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Yang-Baxter Equation

- Starting with a quantum R-matrix, i.e. a particular solution of the Yang-Baxter equation

$$R_{12}(\lambda-\mu)R_{13}(\lambda-\nu)R_{23}(\mu-\nu) = R_{23}(\mu-\nu)R_{13}(\lambda-\nu)R_{12}(\lambda-\mu)$$

- one obtains the L-operator corresponding to each site of the chain

$$L_{0a}(\lambda - z_a) = R_{0a}(\lambda - z_a)$$

- the corresponding T-matrix

$$T(\lambda; \{z_a\}_1^N) = L_{0N}(\lambda - z_N)\dots L_{01}(\lambda - z_1) = \overleftarrow{\prod_{a=1}^{N}} L_{0a}(\lambda - z_a)$$

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RTT-relations and ABA

- Faddeev-Reshetikhin-Takhtajan (FRT) relations

$$R_{12}(\lambda-\mu)T_1(\lambda)T_2(\mu) = T_2(\mu)T_1(\lambda)R_{12}(\lambda-\mu)$$

- transfer matrix

$$t(\lambda) = \operatorname{tr} T(\lambda; \{z_a\}_1^N)$$

  generates an Abelian subalgebra $t(\lambda)t(\mu) = t(\mu)t(\lambda)$

- Algebraic Bethe Ansatz, spectrum, Bethe vectors

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sl_2-invariant r-matrix

Using the standard sl_2 generators $(h, X^{\pm})$

$$[h, X^{\pm}] = \pm 2X^{\pm}, \qquad [X^+, X^-] = h,$$

and the quadratic tensor Casimir of sl_2

$$c_2^{\otimes} = h \otimes h + 2\left(X^+ \otimes X^- + X^- \otimes X^+\right)$$

one can write the sl_2-invariant r-matrix

$$r(\lambda - \mu) = \frac{c_2^{\otimes}}{\lambda - \mu}.$$

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sl_2 r-matrix with a Jordanian deformation

The sl_2-invariant r-matrix with an extra Jordanian term is

$$r_\xi^J(\mu,\nu) = \frac{c_2^{\otimes}}{\mu-\nu} + \xi(h \otimes X^+ - X^+ \otimes h).$$

It can be obtained as the semi-classical limit of the Yang R-matrix twisted by the Jordanian twist element

$$\mathcal{F} = \exp(h \otimes \ln(1+\theta X^+)) \in U(sl(2)) \otimes U(sl(2))$$

which satisfies the Drinfeld twist equation.

Deformed sl_2 r-matrix

We will consider the sl_2-invariant r-matrix with a deformation depending on the spectral parameters

$$r_\xi(\lambda,\mu) = \frac{c_2^{\otimes}}{\lambda-\mu} + \xi\left(h\otimes(\mu X^+) - (\lambda X^+)\otimes h\right).$$

The matrix form of $r_\xi(\lambda,\mu)$ in the fundamental representation of sl_2 is given explicitly by

$$r_\xi(\lambda,\mu) = \begin{pmatrix} \frac{1}{\lambda-\mu} & \mu\xi & -\lambda\xi & 0 \\ 0 & -\frac{1}{\lambda-\mu} & \frac{2}{\lambda-\mu} & \lambda\xi \\ 0 & \frac{2}{\lambda-\mu} & -\frac{1}{\lambda-\mu} & -\mu\xi \\ 0 & 0 & 0 & \frac{1}{\lambda-\mu} \end{pmatrix},$$

here $\lambda,\mu\in\mathbb{C}$ are the so-called spectral parameters and $\xi\in\mathbb{C}$ is a deformation parameter.

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L-operator

The next step is to introduce the L-operator of the Gaudin model

$$L(\lambda) = \begin{pmatrix} h(\lambda) & 2X^-(\lambda) \\ 2X^+(\lambda) & -h(\lambda) \end{pmatrix}$$

the entries are given by

$$h(\lambda) = \sum_{a=1}^{N} \left(\frac{h_a}{\lambda - z_a} + \xi z_a X_a^+ \right),$$

$$X^-(\lambda) = \sum_{a=1}^{N} \left(\frac{X_a^-}{\lambda - z_a} - \frac{\xi}{2}\lambda h_a \right), \quad X^+(\lambda) = \sum_{a=1}^{N} \frac{X_a^+}{\lambda - z_a},$$

with $h_a = \pi_a^{(\ell_a)}(h) \in End(V_a^{(\ell_a)}), X_a^{\pm} = \pi_a^{(\ell_a)}(X^{\pm}) \in End(V_a^{(\ell_a)})$

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and $\pi_a^{(\ell_a)}$ is an irreducible representation of sl_2 whose representation space is $V_a^{(\ell_a)}$ corresponding to the highest weight ℓ_a and the highest weight vector $\omega_a \in V_a^{(\ell_a)}$, i.e.

$$X_a^+ \omega_a = 0 \quad \text{and} \quad h_a \omega_a = \ell_a \omega_a,$$

at each site $a = 1, \ldots, N$. Notice that ℓ_a is a nonnegative integer and the $(\ell_a + 1)$-dimensional representation space $V_a^{(\ell_a)}$ has the natural Hermitian inner product such that

$$(X_a^+)^* = X_a^-, \quad (X_a^-)^* = X_a^+ \quad \text{and} \quad h_a^* = h_a.$$

The space of states of the system $\mathcal{H} = V_1^{(\ell_1)} \otimes \cdots \otimes V_N^{(\ell_N)}$ is naturally equipped with the Hermitian inner product $\langle \cdot | \cdot \rangle$ as a tensor product of the spaces $V_a^{(\ell_a)}$ for $a = 1, \ldots, N$.

Sklyanin Linear Bracket

The L-operator satisfies the so-called Sklyanin linear bracket

$$\left[\underset{1}{L}(\lambda), \underset{2}{L}(\mu)\right] = -\left[r_\xi(\lambda,\mu)\, , \, \underset{1}{L}(\lambda) + \underset{2}{L}(\mu)\right] .$$

Both sides of this relation have the usual commutators of the 4×4 matrices $\underset{1}{L}(\lambda) = L(\lambda) \otimes \mathbb{1}$, $\underset{2}{L}(\mu) = \mathbb{1} \otimes L(\mu)$ and $r_\xi(\lambda, \mu)$, where $\mathbb{1}$ is the 2×2 identity matrix.

Gaudin Algebra

The relation above is a compact matrix form of the following commutation relations

$$
\begin{aligned}
[h(\lambda), h(\mu)] &= 2\xi\left(\lambda X^+(\lambda) - \mu X^+(\mu)\right) \\
\left[X^-(\lambda), X^-(\mu)\right] &= -\xi\left(\mu X^-(\lambda) - \lambda X^-(\mu)\right), \\
\left[X^+(\lambda), X^-(\mu)\right] &= -\frac{h(\lambda) - h(\mu)}{\lambda - \mu} + \xi\mu X^+(\lambda), \\
\left[X^+(\lambda), X^+(\mu)\right] &= 0, \\
\left[h(\lambda), X^-(\mu)\right] &= 2\frac{X^-(\lambda) - X^-(\mu)}{\lambda - \mu} + \xi\mu h(\mu), \\
\left[h(\lambda), X^+(\mu)\right] &= -2\frac{X^+(\lambda) - X^+(\mu)}{\lambda - \mu}.
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$$

Gaudin Algebra

In order to define a dynamical system besides the algebra of observables a Hamiltonian should be specified. Due to the Sklyanin linear bracket the generating function

$$t(\lambda) = \frac{1}{2}\,\mathrm{tr} L^2(\lambda) = h^2(\lambda) - 2h'(\lambda) + 2\left(2X^-(\lambda) + \xi\lambda\right) X^+(\lambda)$$

satisfies

$$t(\lambda)t(\mu) = t(\mu)t(\lambda).$$

The pole expansion of the generating function $t(\lambda)$ is

$$t(\lambda) = \sum_{a=1}^{N}\left(\frac{\ell_a(\ell_a+2)}{(\lambda-z_a)^2} + \frac{2H^{(a)}}{\lambda-z_a}\right) + 2\xi(1-h_{(gl)})X^+_{(gl)} + \xi^2 \sum_{a,b=1}^{N} z_a z_b X_a^+ X_b^+ .$$

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$$t(\lambda) = \sum_{a=1}^{N} \left(\frac{\ell_a(\ell_a+2)}{(\lambda - z_a)^2} + \frac{2H^{(a)}}{\lambda - z_a} \right) + 2\xi(1 - h_{(gl)}) X^+_{(gl)} + \xi^2 \sum_{a,b=1}^{N} z_a z_b X^+_a X^+_b .$$

Gaudin Model

The residues of the generating function $t(\lambda)$ at the points $\lambda = z_a$, $a = 1, \ldots, N$ are the Gaudin Hamiltonians

$$H^{(a)} = \sum_{b \neq a}^{N} \left(\frac{c_2(a,b)}{z_a - z_b} + \xi \left(z_b h_a X_b^+ - z_a h_b X_a^+ \right) \right),$$

where $c_2(a,b) = h_a h_b + 2(X_a^+ X_b^- + X_a^- X_b^+)$. In the constant term of the pole expansion the notation

$$Y_{(gl)} = \sum_{a=1}^{N} Y_a,$$

for $Y = (h, X^{\pm})$, was used to denote the generators of the so-called global sl_2 algebra. In the case when $\xi = 0$ the global sl_2 algebra is a symmetry of the system.

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Gaudin Model

Finally, it is important to notice the following relation

$$t(\lambda) = t(\lambda)_0 + 2\xi \left(h(\lambda)_0 \hat{X}^+_{(gl)} + X^+_{(gl)} - \lambda h_{(gl)} X^+(\lambda) \right) + \xi^2 (\hat{X}^+_{(gl)})^2,$$

where $\hat{X}^+_{(gl)} = \sum_{a=1}^N z_a X^+_a$, $h(\lambda)_0 = h(\lambda)|_{\xi=0}$ and $t(\lambda)_0 = t(\lambda)|_{\xi=0}$ is the generating function of the integrals of motion in the sl_2-invariant case.

Outline

Highest Spin Vector Ω_+

In the space of states $\mathcal{H}$ the vector

$$\Omega_+ = \omega_1 \otimes \cdots \otimes \omega_N$$

is such that $\langle \Omega_+ | \Omega_+ \rangle = 1$ and

$$X^+(\lambda)\Omega_+ = 0, \quad h(\lambda)\Omega_+ = \rho(\lambda)\Omega_+,$$

with

$$\rho(\lambda) = \sum_{a=1}^{N} \frac{\ell_a}{\lambda - z_a}.$$

Creation Operators

The creation operators used in the sl_2-invariant Gaudin model coincide with one of the L-matrix entry. However, in the present case these operators are non-homogeneous polynomials of the operator $X^-(\lambda)$. It is convenient to define a more general set of operators.

Given integers M and $k \geq 0$, let $\mu = \{\mu_1, \ldots, \mu_M\}$ be a set of complex scalars. Define the operators

$$B_M^{(k)}(\mu) = \prod_{\substack{n=k \\ \rightarrow}}^{M+k-1} \left(X^-(\mu_{n-k+1}) + n\xi\mu_{n-k+1}\right),$$

with $B_0^{(k)} = 1$ and $B_M^{(k)} = 0$ for $M < 0$.

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Creation Operators

The commutation relations between the operators $h(\lambda)$, $X^{\pm}(\lambda)$ and the $B_M^{(k)}(\mu_1, \dots, \mu_M)$ operators are given by

$$h(\lambda)B_M^{(k)}(\boldsymbol{\mu}) = B_M^{(k)}(\boldsymbol{\mu})h(\lambda) + 2\sum_{i=1}^{M} \frac{B_M^{(k)}(\lambda \cup \boldsymbol{\mu}^{(i)}) - B_M^{(k)}(\boldsymbol{\mu})}{\lambda - \mu_i}$$

$$+ \xi \sum_{i=1}^{M} B_{M-1}^{(k+1)}(\boldsymbol{\mu}^{(i)}) \left(\mu_i \hat{\beta}_M(\mu_i; \boldsymbol{\mu}^{(i)}) - 2k \right);$$

Creation Operators

$$X^+(\lambda)B_M^{(k)}(\boldsymbol{\mu}) = B_M^{(k)}(\boldsymbol{\mu})X^+(\lambda) - 2\sum_{\substack{i,j=1\\i<j}}^{M} \frac{B_{M-1}^{(k+1)}(\lambda \cup \boldsymbol{\mu}^{(i,j)})}{(\lambda-\mu_i)(\lambda-\mu_j)}$$

$$- \sum_{i=1}^{M} B_{M-1}^{(k+1)}(\boldsymbol{\mu}^{(i)}) \left(\frac{\hat{\beta}_M(\lambda;\boldsymbol{\mu}^{(i)}) - \hat{\beta}_M(\mu_i;\boldsymbol{\mu}^{(i)})}{\lambda-\mu_i} - \xi\mu_i X^+(\lambda) \right);$$

$$X^-(\lambda)B_M^{(k)}(\boldsymbol{\mu}) = B_{M+1}^{(k)}(\lambda\cup\boldsymbol{\mu}) - \xi\sum_{i=1}^{M}\mu_i B_M^{(k)}(\lambda\cup\boldsymbol{\mu}^{(i)}).$$

Creation Operators

The notation used above is the following. Let $\boldsymbol{\mu} = \{\mu_1, \ldots, \mu_M\}$ be a set of complex scalars, then

$$\boldsymbol{\mu}^{(i_1,\ldots,i_k)} = \boldsymbol{\mu} \setminus \{\mu_{i_1}, \ldots, \mu_{i_k}\}$$

for any distinct $i_1, \ldots, i_k \in \{1, \ldots, M\}$.
It is important to notice that the creation operators that yield the Bethe states of the system are the operators $B_M^{(0)}(\boldsymbol{\mu})$, below denoted by $B_M(\boldsymbol{\mu})$.
A recursive relation defining the creation operators is

$$B_M(\boldsymbol{\mu}) = B_{M-1}(\boldsymbol{\mu}^{(M)}) \left(X^-(\mu_M) + (M-1)\xi\mu_M\right).$$

Creation Operators

The commutation relations between the generating function of the integrals of motion $t(\lambda)$ and the B-operators are given by

$$
\begin{aligned}
t(\lambda)B_M(\boldsymbol{\mu}) = B_M(\boldsymbol{\mu}) & \left(t(\lambda) - \sum_{i=1}^{M} \frac{4h(\lambda)}{\lambda - \mu_i} + \sum_{i<j}^{M} \frac{8}{(\lambda - \mu_i)\left(\lambda - \mu_j\right)} + 4M\xi\lambda\, X^+(\lambda) \right) \\
& + 4 \sum_{i=1}^{M} \frac{B_M(\lambda \cup \boldsymbol{\mu}^{(i)})}{\lambda - \mu_i} \hat{\beta}_M(\mu_i; \boldsymbol{\mu}^{(i)}) \\
& + 2\xi \sum_{i=1}^{M} B^{(1)}_{M-1}(\boldsymbol{\mu}^{(i)})(\mu_i h(\lambda) + 1)\hat{\beta}_M(\mu_i; \boldsymbol{\mu}^{(i)}) \\
& + 4\xi \sum_{\substack{i,j=1 \\ i\neq j}}^{M} \mu_i \frac{B^{(1)}_{M-1}(\lambda \cup \boldsymbol{\mu}^{(i,j)}) - B^{(1)}_{M-1}(\boldsymbol{\mu}^{(i)})}{\lambda - \mu_j} \hat{\beta}_M(\mu_i; \boldsymbol{\mu}^{(i)}) \\
& + \xi^2 \sum_{\substack{i,j=1 \\ i\neq j}}^{M} \mu_i B^{(2)}_{M-2}(\boldsymbol{\mu}^{(i,j)}) \left(\mu_j \hat{\beta}_{M-1}(\mu_j; \boldsymbol{\mu}^{(i,j)}) - 2 \right) \hat{\beta}_M(\mu_i; \boldsymbol{\mu}^{(i)}) \\
& + 2\xi^2 \sum_{i=1}^{M} \mu_i^2 B^{(1)}_{M-1}(\boldsymbol{\mu}^{(i)}) X^+(\mu_i). \qquad \text{(III.1)}
\end{aligned}
$$

Spectrum and Bethe vectors of the mode

The highest spin vector Ω_+ is an eigenvector of the operator $t(\lambda)$

$$t(\lambda)\Omega_+ = \left(h^2(\lambda) - 2h'(\lambda) + 2\left(2X^-(\lambda) + \xi\lambda\right)X^+(\lambda)\right)\Omega_+ = \Lambda_0(\lambda)\Omega_+$$

with the corresponding eigenvalue

$$\Lambda_0(\lambda) = \rho^2(\lambda) - 2\rho'(\lambda) = \sum_{a=1}^{N}\frac{2}{\lambda - z_a}\left(\sum_{b\neq a}^{N}\frac{\ell_a\ell_b}{z_a - z_b}\right) + \sum_{a=1}^{N}\frac{\ell_a(\ell_a+2)}{(\lambda - z_a)^2}.$$

Spectrum and Bethe Vectors of the Mode

Furthermore, the action of the B-operators on the highest spin vector Ω_+ yields the Bethe vectors

$$\Psi_M(\boldsymbol{\mu}) = B_M(\boldsymbol{\mu})\Omega_+,$$

so that

$$\begin{aligned} t(\lambda)\Psi_M(\boldsymbol{\mu}) &= t(\lambda)B_M(\boldsymbol{\mu})\Omega_+ = \Lambda_0(\lambda)\Psi_M(\boldsymbol{\mu}) + \left[t(\lambda), B_M(\boldsymbol{\mu})\right]\Omega_+, \\ &= \Lambda_M(\lambda;\boldsymbol{\mu})\Psi_M(\boldsymbol{\mu}) \end{aligned}$$

with the eigenvalues

$$\Lambda_M(\lambda;\boldsymbol{\mu}) = \rho_M^2(\lambda;\boldsymbol{\mu}) - 2\frac{\partial\rho_M}{\partial\lambda}(\lambda;\boldsymbol{\mu}) \quad \text{and} \quad \rho_M(\lambda;\boldsymbol{\mu}) = \rho(\lambda) - \sum_{i=1}^{M}\frac{2}{\lambda-\mu_i},$$

Spectrum and Bethe vectors of the mode

provided that the Bethe equations are imposed on the parameters $\boldsymbol{\mu} = \{\mu_1, \ldots, \mu_M\}$

$$\rho_M(\mu_i; \boldsymbol{\mu}^{(i)}) = \sum_{a=1}^{N} \frac{\ell_a}{\mu_i - z_a} - \sum_{j \neq i}^{M} \frac{2}{\mu_i - \mu_j} = 0, \quad i = 1, \ldots, M.$$

The Bethe vectors $\Psi_M(\boldsymbol{\mu})$ are eigenvectors of the Gaudin Hamiltonians

$$H^{(a)} \Psi_M(\boldsymbol{\mu}) = E_M^{(a)} \Psi_M(\boldsymbol{\mu}),$$

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Outline

1. Introduction
   - Quantum Integrable Systems
   - Gaudin Models
   - Quantum Inverse Scattering Method
2. Deformed Gaudin Model
   - Classical r-matrix
   - Sklyanin Bracket and Gaudin Algebra
   - Algebraic Bethe Ansatz
3. Conclusions
   - Summary
   - Outlook

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- The Gaudin model based on the deformed sl_2 r-matrix is studied.
- The usual Gaudin realization of the model is introduced and the B-operators $B_M^{(k)}(\mu_1, \ldots, \mu_M)$ are defined as non-homogeneous polynomials of the operator $X^-(\lambda)$.
- These operators are symmetric functions of their arguments and they satisfy certain recursive relations with explicit dependency on the quasi-momenta $\mu_1, \ldots, \mu_M$.
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- The commutator of the creation operators with the generating function of the Gaudin model under study is calculated explicitly.
- Based on the previous result the spectrum of the system is determined.
- It turns out that the spectrum of the system and the corresponding Bethe equations coincide with the ones of the sl_2-invariant model!
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- The explicit form of the generalized Bethe vectors associated to the Jordan canonical form of the generating function $t(\lambda)$ remains an open problem.
- The well known relation between the off-shell Bethe vectors of the Gaudin models related to simple Lie algebras and the solutions of Knizhnik-Zamolodchikov equation also holds for the KZ equation related to the sl_2 classical r-matrix with the jordanian twist. However, in the present case the relation between the Bethe vectors and the solutions of the corresponding KZ is yet to be established.

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