Two-group formulation of General Relativity and state-sum models

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We show that ...

1. Introduction

GR was originally formulated as a dynamical theory of metrics on a spacetime manifold, and it turned out that for a non-perturbative quantization it is more advantegous to reformulate it as a theory of connections, see [1].

More precisely, GR can be represented as a constrained BF theory, see [2], and this approach led to formulation of spin foam models of quantum GR, see [3, 1]. The EPRL/FK class of spin foam models [4, 5] allows for a construction of finite QG transition amplitudes [8, 9, 10] and the corresponding classical limit is GR [11]. However, the absence of the tetrads from the theory makes it difficult to couple massive fermions [6] as well as the gauge fields [7], so that there is a need for a BF-type reformulation of GR which will include the tetrads.

One way to do this is to introduce the cosmological constant and represent GR as a BF theory for the AdS/dS group with a symmetry breaking term [12]. However, the cosponding spin foam perturbation theory is difficult to formulate [13], since the symmetry breaking term is the perturbation, and there is no efficient mathematical formalism to calculate the corrections.

Another possible approach is to use the Poincare group, since GR can be represented as a gauge theory for the Poincare group, see [14]. However, the transformation law for the tetrads under the local translations does not coincide with the diffeomorphism transformations. Although the diffeomorphisms transformations of the tetrads can be represented as suitably restricted local translations, it is not clear how to implement

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this restriction in the corresponding BF-theory type reformulation of the Einstein-Cartan action.

In this paper we will show that this problem can be resolved by using the BF theory for 2-groups, also known as BFCG theory, see [15, 16]. The 2-groups are category theory generalizations of the usual groups and the Poincare group can be naturally embedded into a 2-group, see [17]. We will show that GR can be represented as a constrained BFCG theory for the Poincare 2-group (in analogy with the result that GR is the constrained BF theory for the Lorentz group).

2. Poincare 2-group and GR

One way to generalize the notion of the group is to use the category theory. A category consists of objects and maps between the objects (called morphisms) such that natural composition rules between the morphisms are satisfied, see [17]. A 2-category consists of objects, morphisms and maps between morphisms (called 2-morphisms) such that natural composition rules are satisfied. A group is then a category with one object where all morphisms are invertible. Similarly, a two-group is a 2-category with one object where all morphisms are invertible. This abstract definition leads to a concrete realization of a 2-group which is given by a crossed module $(G, H, \partial, \triangleright)$. This is a pair of groups G and H, such that $\partial : H \to G$ is a homomorphism and $\triangleright$ is an action of G on H such that certain properties are satisfied, which are direct consequences of the categorical structure, see [17]. A canonical example of a 2-group relevant for physics is the Poincare 2-group, where $G = SO(1,3)$, $H = \mathbf{R}^4$, ∂ is a trivial homomorphism and $\triangleright$ is the usual action of the Lorentz transformations on the $\mathbf{R}^4$ space. The Lorentz group is the group of morphisms, while the usual Poincare group is the group of 2-morphisms.

One can construct a gauge theory on a 4-manifold M based on a crossed module $(G, H, \partial, \triangleright)$ of Lie groups by using one-forms A, which take values in the Lie algebra $\mathbf{g}$ of G, and 2-forms β, which take values in the Lie algebra $\mathbf{h}$ of H [15, 16]. The forms A and β transform under the usual gauge transformations $g : M \to G$ as

$$A \to g^{-1}Ag + g^{-1}dg\,, \quad \beta \to g^{-1} \triangleright \beta\,, \tag{1}$$

while the gauge transformations generated by H are given by

$$A \to A + \partial\eta\,, \quad \beta \to \beta + d\eta + A \wedge^{\triangleright} \eta + \eta \wedge \eta\,, \tag{2}$$

where η is a one-form taking values in $\mathbf{h}$, see [16]. When the group H is abelian, which happens in the Poincare 2-group case, then the $\eta \wedge \eta$ term in (2) vanishes, and one obtains the gauge transformations given in [15] .

The pair (A, β) represents a 2-connection on the 2-bundle associated to the 2-Lie group (G, H) and the manifold M. The corresponding curvature forms are given by

$$\mathcal{F} = dA + A \wedge A - \partial\beta\,, \quad \mathcal{G} = d\beta + A \wedge^{\triangleright} \beta\,, \tag{3}$$

and they transform as

$$\mathcal{F} \mapsto g^{-1}\mathcal{F}g\,, \quad \mathcal{G} \to g^{-1} \triangleright \mathcal{G}\,, \tag{4}$$

under the usual gauge transformations, while

$$\mathcal{F} \to \mathcal{F}\,, \quad \mathcal{G} \to \mathcal{G} + \mathcal{F} \wedge^{\triangleright} \eta\,, \tag{5}$$

under the H-gauge transformations.

One can introduce a natural topological gauge theory determined by the vanishing of the 2-curvature

$$\mathcal{F} = 0\,, \quad \mathcal{G} = 0\,. \tag{6}$$

These equations can be obtained from the action

$$S_0 = \int_M \langle B \wedge \mathcal{F} \rangle_{\mathbf{g}} + \langle C \wedge \mathcal{G} \rangle_{\mathbf{h}}\,, \tag{7}$$

where B is a 2-form taking values in $\mathbf{g}$, C is a one-form taking values in $\mathbf{h}$, $\langle\,,\rangle_{\mathbf{g}}$ is a G-invariant non-degenerate bilinear form in $\mathbf{g}$ and $\langle\,,\rangle_{\mathbf{h}}$ is a G-invariant non-degenerate bilinear form in $\mathbf{h}$. The action (7) is called BFCG action, in analogy with the BF theory action. The gauge transformations of the Lagrange multiplier fields are given by

$$B \to g^{-1} B g\,, \quad C \mapsto g^{-1} \triangleright C\,, \tag{8}$$

for the usual gauge transformations, while

$$B \to B - [C, \eta]\,, \quad C \mapsto C\,, \tag{9}$$

for the H-gauge transformations.

Let us now examine the case of the Poincare 2-group. In this case $A = \omega^{ab} J_{ab}$, $\beta = \beta^a P_a$, where

J are the generators of the Lorentz group, while P are the generators of $\mathbf{R}^4$. Consequently

$$\mathcal{F} = (d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}) J_{ab} = R^{ab} J_{ab}\,, \quad \mathcal{G} = (d\beta^a + \omega^a{}_b \wedge \beta^b) P_a = \nabla \beta^a P_a\,. \tag{10}$$

The G-gauge transformations are the local Lorentz rotations

$$\omega \to g^{-1} \omega g + g^{-1} dg\,, \quad \beta \to g^{-1} \triangleright \beta\,, \tag{11}$$

while the H-gauge transformations are the local translations

$$\delta_\varepsilon \omega = 0\,, \quad \delta_\varepsilon \beta^a = d\varepsilon^a + \omega^a_b \wedge \varepsilon^b\,, \tag{12}$$

where $\eta = \varepsilon^a P_a$.

The BFCG action then becomes

$$S_0 = \int_M (B^{ab} \wedge R_{ab} + C_a \wedge \nabla \beta^a)\,, \tag{13}$$

where

$$\delta_\varepsilon B = 0\,, \quad \delta_\varepsilon C = 0\,. \tag{14}$$

Note that the transformation properties of the one-forms C^a are the same as the transformation properties of the tetrad one forms e^a under the local Lorentz and the diffeomorphism transformations. Hence one can identify the C fields with the tetrads and we write

$$S_0 = \int_M (B^{ab} \wedge R_{ab} + e^a \wedge \nabla\beta_a)\,. \tag{15}$$

The action (15) gives a theory of flat metrics, since $R^{ab} = 0$ implies the vanishing of the Reimann tensor. In order to obtain GR, we need that the Ricci tensor vanishes. In the BF theory approach to GR, this problem is resolved by constraining the B field, such that $B^{ab} = \varepsilon^{abcd} e_c \wedge e_d$. Since in the 2-group formulation the tetrads are explicitly present, the required constraint is simply

$$B^{ab} = \varepsilon^{abcd} e_c \wedge e_d\,. \tag{16}$$

Hence the action for GR in the 2-group approach is given by

$$S = \int_M \left(B^{ab} \wedge R_{ab} + e^a \wedge \nabla\beta_a - \phi_{ab} \wedge (B^{ab} - \varepsilon^{abcd} e_c \wedge e_d) \right)\,. \tag{17}$$

The equations of motion are

$$\begin{aligned} R_{ab} - \phi_{ab} &= 0 && (18)\\ \nabla\beta_a + 2\varepsilon_{abcd}\phi^{bc} \wedge e^d &= 0 && (19)\\ \nabla B_{ab} - e_{[a} \wedge \beta_{b]} &= 0 && (20)\\ \nabla e_a &= 0 && (21)\\ B_{ab} - \varepsilon_{abcd} e^c \wedge e^d &= 0\,. && (22) \end{aligned}$$

From $B = \varepsilon e \wedge e$ it follows that $\nabla B \propto \varepsilon e \wedge \nabla e$, so that $\nabla B = 0$ due to (21). The equation (20) then implies that $e_{[a} \wedge \beta_{b]} = 0$. For invertible tetrads we then obtain $\beta = 0$, so that (18) and (19) imply

$$\varepsilon_{abcd} R^{bc} \wedge e^d = 0\,. \tag{23}$$

The equation (23) is the equation of motion for the EC action

$$S_{EC} = \int_M \varepsilon^{abcd} e_a \wedge e_b \wedge R_{cd}\,, \tag{24}$$

for the e variations, while (21) is equivalent to $\delta S_{EC}/\delta\omega$ when the tetrads are invertible.

3. Coupling of matter

Since the tetrads are present in the BFCG action, the coupling of matter fields is essentially given by the coupling of matter fields in the EC formulation. The only subtlety is in the coupling of fermions, since their presence introduces a non-zero torsion.

The Dirac action for the fermion field in the EC formulation is given by

$$S_D = i\kappa_1 \int \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \bar\psi \left(\gamma^d \overleftrightarrow{d} + \{\omega, \gamma^d\} + \frac{im}{2} e^d \right) \psi\,, \tag{25}$$

where $\omega = \omega_{ab}[\gamma^a, \gamma^b]/8$ and $\kappa_1 = 8\pi l_p^2/3$. The $\delta(S_{EC} + S_D)/\delta\omega$ equation gives the torsion $T_a \equiv \nabla e_a = -\kappa_2 s_a$, where

$$s_a = i\varepsilon_{abcd}\, e^b \wedge e^c\, \bar\psi \gamma_5 \gamma^d \psi\,,$$

is the spin 2-form, and $\kappa_2 = -3\kappa_1/4$. Hence in the BFCG formulation we need a term $\int_M \beta_a \wedge s^a$ in the action.

Let us consider the action

$$S_m = S + S_D + S_{\beta\psi}\,, \tag{26}$$

where

$$S_{\beta\psi} = i\kappa_2 \int \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c\, \bar\psi \gamma_5 \gamma^d \psi\,.$$

The equations of motion are

$$\begin{aligned}
\frac{\delta S}{\delta B^{ab}} &= R_{ab} - \phi_{ab} = 0 & (27)\\
\frac{\delta S}{\delta e^a} &= -\nabla\beta_a - \varepsilon_{abcd} e^b \wedge \left[2\phi^{cd} - \frac{3i\kappa_1}{2}\beta^c \bar\psi\gamma_5\gamma^d\psi + 3i\kappa_1 e^c \wedge \bar\psi \left(\gamma^d \overrightarrow{\nabla} - \overleftarrow{\nabla}\gamma^d + \frac{im}{6}e^d\right)\psi\right] = 0, & (28)\\
\frac{\delta S}{\delta \omega^{ba}} &= \nabla B_{ab} - e_{[a} \wedge \beta_{b]} - 2\kappa_2 \varepsilon_{abcd} e^c \wedge s^d = 0\,, & (29)\\
\frac{\delta S}{\delta \beta^a} &= \nabla e_a + \kappa_2 s_a = 0\,, & (30)\\
\frac{\delta S}{\delta \phi^{ba}} &= B_{ab} - \varepsilon_{abcd} e^c \wedge e^d = 0\,, & (31)\\
\frac{\delta S}{\delta \bar\psi} &= i\kappa_1 \varepsilon_{abcd} e^a \wedge e^b \wedge \left(2e^c \wedge \gamma^d \nabla + \frac{im}{2} e^c \wedge e^d - 3(\nabla e^c)\gamma^d - \frac{3}{4}\beta^c\gamma_5\gamma^d\right)\psi = 0\,, & (32)
\end{aligned}$$

From the $\delta/\delta\phi$ equation it follows that $\nabla B \propto \varepsilon e \wedge \nabla e$, so that the $\delta/\delta\omega$ equation gives

$$2\,\varepsilon_{abcd} e^c \wedge \left(\nabla e^d + \kappa_2 s^d\right) + e_{[a} \wedge \beta_{b]} = 0$$

which gives $e_{[a} \wedge \beta_{b]} = 0$, since the $\delta/\delta\beta$ equation is $\nabla e + \kappa_2 s = 0$. If the tetrads are invertible, one then obtains $\beta^a = 0$, so that the $\delta/\delta e$ equation gives $\varepsilon e \wedge (R - T_\psi) = 0$, where T_ψ is the energy momentum 2-form of the fermions.

The $\delta/\delta\psi$ and $\delta/\delta\bar\psi$ equations are related by spinor conjugation. For the invertible tetrads, using $\nabla e = -\kappa_2 s$, the $\delta/\delta\bar\psi$ equation reduces to the usual Dirac equation

$$(i\gamma^\mu \nabla_\mu - m)\,\psi = 0\,, \tag{33}$$

where $\gamma^\mu = e^\mu{}_a \gamma^a$.

As far as the scalar and YM fields are concerned, they do not couple to ω so that one simply adds the corresponding EC formalism terms to S_m

$$S_m \to S_m + \int_M |e|\,(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + g^{\mu\nu}g^{\rho\sigma}\, Tr\, F_{\mu\rho}F_{\nu\sigma})\, d^4x\,, \tag{34}$$

where $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$.

One can also introduce the Immirzi parameter γ, by adding an additional term S_γ to the action S_m, where

$$S_\gamma = -\frac{1}{\gamma}\int \phi^{ab} \wedge e_a \wedge e_b + \frac{i\kappa_2}{\gamma^2+1}\int \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c \bar\psi\gamma_5\gamma^d\psi + \frac{i\kappa_2\gamma}{\gamma^2+1}\int e^a \wedge e^b \wedge \beta_a \bar\psi \gamma_5 \gamma_b \psi\,.$$

The resulting equations of motion are equivalent to the equations of motion obtained from the action $S_{EC} + S_D + S_H$, where S_H is the Holst term [18]

$$S_H = -\frac{2}{\gamma}\int e^a \wedge e^b \wedge R_{ab}\,.$$

The physical motivation for the introduction of the Immirzi parameter lies in the fact that it is the coupling constant between fermions and torsion, as discussed in detail in [19, 20].

4. State-sum models

Given the BFCG form of the EC action, one can now proceed to quantize the theory by using the same approach as in the case of spin foam models: first formulate the state-sum model for the topological theory given by the action for the unconstrained BFCG theory, and after that impose the constraint $B = \varepsilon e \wedge e$.

In the topological case one starts from the path-integral

$$\begin{aligned} Z &= \int \mathcal{D}A\,\mathcal{D}\beta\,\mathcal{D}B\,\mathcal{D}C\,\exp\left(i\int_M (\langle B\wedge\mathcal{F}\rangle + \langle C\wedge\mathcal{G}\rangle)\right) \\ &= \int \mathcal{D}A\,\mathcal{D}\beta\,\delta(\mathcal{F})\,\delta(\mathcal{G})\,, \end{aligned} \tag{35}$$

see [15].

Let T be a regular triangulation of M, and T^* the dual tringulation, then

$$Z = \int\prod_l dg_l \int\prod_f dh_f \prod_f \delta(g_f)\prod_p \delta(h_p)\,, \tag{36}$$

where l, f and p denote the 1,2 and 3-cells of T^*, respectivelly, and one has

$$g_l = \exp\left(\int_l A\right)\,,\quad h_f = \exp\left(\int_f \beta\right)\,.$$

The group elements $g_l \in G$ and $h_f \in H$ represent the corresponding one and 2-holonomies of A and β, respectively. The group element $g_f = \prod_{l\in\partial f} g_l$ is the holonomy along the boundary of f and one has $g_f \approx e^{\int_f \mathcal{F}}$, while h_p is the 2-holonomy alonog the closed surface ∂p, and it is given by

$$h_p = \prod_{f\in\partial p}\tilde{h}_f \approx \exp(\int_p \mathcal{G})\,,$$

where some of the $\tilde{h}_f$ are given by $g_l \triangleright h_f$, where $l \in p$ and $l \notin \partial f$, while the other $\tilde{h}_f$ are equal to h_f, see [15] for the case when p is a tetrahedron and see [21] when p is an arbitrary polyhedron.

In the case of the Poincare 2-group (36) can be written as

$$Z = \int\prod_l dg_l \int\prod_f d^4\vec{x}_f \prod_f \delta(g_f)\prod_p \delta(\vec{x}_p)\,, \tag{37}$$

where $\vec{x}_p = \vec{x}_f + ... + g_l \vec{x}_{f'}$ and $f, ..., f' \in \partial p$. The Lorentz group delta function can be expanded by using the Plancherel teorem

$$\delta(g_f) = \sum_{\Lambda_f} d\mu(\Lambda_f)\, \chi(g_f, \Lambda_f)\,,$$

where $\Lambda = (j, \rho)$ are the unitary irreducible representations, and $d\mu$ is the appropriate integration measure, see [**?**], while

$$\delta(\vec{x}_p) = \frac{1}{(2\pi)^4} \int_{\mathbf{R}^4} d^4 \vec{L}_p \exp\left(i\vec{x}_p \cdot \vec{L}_p \right)\,.$$

For the sake of simplicity, let us consider the Euclidean case, so that the Poincare 2-group becomes the Euclidean 2-group. The Lorentz group is then replaced by the $SO(4)$ group and $\Lambda = (j^+, j^-)$ is a pair of $SU(2)$ spins, so that

$$Z = \sum_{\Lambda_f} \int \prod_p d^4 \vec{L}_p \int \prod_l dg_l \prod_f d^4 \vec{x}_f \dim \Lambda_f\, \chi(\Lambda_f, g_f) \prod_p e^{i\vec{x}_p \cdot \vec{L}_p}\,. \tag{38}$$

After integrating $\vec{x}_f$, we obtain

$$\begin{aligned} Z &= \sum_{\Lambda_f} \int \prod_p d^4 \vec{L}_p \int \prod_l dg_l \prod_f \dim \Lambda_f\, \chi(\Lambda_f, g_f) \\ &\qquad \prod_f \delta\left(g_{l_1(p_1,f)} \vec{L}_{p_1,f} + g_{l_2(p_2,f)} \vec{L}_{p_2,f} + g_{l_3(p_3,f)} \vec{L}_{p_3,f} \right)\,, \end{aligned} \tag{39}$$

where p_1, p_2 and p_3 are the polyhedra which share the face f, and l_1, l_2 and l_3 are the corresponding dual edges satisfying $l_k \in p_k$ and $l_k \notin f$.

It is instructive to rewrite (39) by using the simplices of $T(M)$

$$\begin{aligned} Z &= \sum_{\Lambda_\Delta} \int \prod_\varepsilon d^4 \vec{L}_\varepsilon \int \prod_\tau dg_\tau \prod_\Delta \dim \Lambda_\Delta\, \chi(\Lambda_\Delta, g_\Delta) \\ &\qquad \prod_\Delta \delta\left(g_{\tau_1(\varepsilon_1,\Delta)} \vec{L}_{\varepsilon_1,\Delta} + g_{\tau_2(\varepsilon_2,f)} \vec{L}_{\varepsilon_2,\Delta} + g_{\tau_3(\varepsilon_3,\Delta)} \vec{L}_{\varepsilon_3,\Delta} \right)\,, \end{aligned} \tag{40}$$

where ε_k are the edges of Δ and $\varepsilon_k \in \tau_k$ but $\Delta \notin \tau_k$. The delta function $\delta(g_1 \vec{L}_1 + g_2 \vec{L}_2 + g_3 \vec{L}_3)$ implies that the norms L_k of the 4-vectors $\vec{L}_k$ satisfy the triangle inequalities. This implies that L_ε can be interpreted as the length of an edge ε.

The state sum (40) can be represented as a categorical state sum for the representations of the Poincare/Euclidean 2-group. These representations were studied in [22, 23], and in order to show this, let us write $\vec{L}_\varepsilon = L_\varepsilon \vec{n}_\varepsilon$, where $\vec{n}$ is a unit four-vector, so that

$$Z = \int \prod_\varepsilon L_\varepsilon^3 dL_\varepsilon \sum_{\Lambda_\Delta, I_\tau} W(L, \Lambda, I)\,, \tag{41}$$

where I_τ is the intertwiner for the four Λ of a tetrahedron and

$$\begin{aligned} \sum_I W(L, \Lambda, I) &= \int \prod_\varepsilon d\Omega_\varepsilon \int \prod_\tau dg_\tau \prod_\Delta \dim \Lambda_\Delta\, \chi(\Lambda_\Delta, g_\Delta) \\ &\qquad \prod_\Delta \delta\left(g_{\tau_1(\varepsilon_1,\Delta)} \vec{L}_{\varepsilon_1,\Delta} + g_{\tau_2(\varepsilon_2,f)} \vec{L}_{\varepsilon_2,\Delta} + g_{\tau_3(\varepsilon_3,\Delta)} \vec{L}_{\varepsilon_3,\Delta} \right)\,, \end{aligned} \tag{42}$$

where $d\Omega$ is the 3-sphere volume measure.

Given that the iretractable representations of the Euclidean 2-group are classified by the positive numbers L, see [23], and that the intertwiner for three L-representations is a pair of $SU(2)$ spins, such that the L satisfy the triangle inequalities, then (41) is exactly a categorical state-sum for these representations. The form of Z in the Poincare case will be the same as (41), but the structure of W will be different due to $\Lambda_\Delta = (j_\Delta, \rho_\Delta)$.

The state sums/integrals in (41) will be divergent, but what is important is the form of the 3-complex amplitude W. Given the form of the topological amplitude, one can then try to implement the constraint $B = \varepsilon e \wedge e$ in order to obtain the state sum for GR, similarly to what was done in the case of spin foam models. Since the labels L_ε can be interpreted as the lengths of the edges of $T(M)$, then the result of the implementation of the constraints should be a reduction

$$\Lambda_\Delta \to j_\Delta \,, \quad I_\tau \to \iota_\tau \,,$$

where j is an $SU(2)$ spin and ι is an $SU(2)$ intertwiner. We then expect to have

$$Z_{GR} = \int \prod_\varepsilon \mu(L_\varepsilon)\, dL_\varepsilon \sum_{j_\Delta, \iota_\tau} W_{GR}(L_\varepsilon, j_\Delta, \iota_\tau)\,, \tag{43}$$

where the amplitude W_{GR} has to be such that Z_{GR} is finite and in the classical limit one obtains GR. For this to happen it is crucial that the large-length asymptotics of W is given by

$$W_{GR}(\lambda L, \lambda j, \iota) \approx N(\lambda)\, \frac{\exp(i\lambda S_R(L))}{\lambda^n}\,, \tag{44}$$

for $\lambda \to \infty$, where N is a homogenous function of order zero, $n > 0$ and S_R is the Regge action, see [11] for the spin-foam case.

Coupling matter in the model (43) will be easier than in the EPRL/FK model, since the edge lenghts L_ε are explicitly present. One can use for the matter amplitudes

$$W_{matt}(L, j) \propto \exp(i S_R^{(matt)})\,,$$

where $S_R^{(matt)}$ is the matter Regge action. The expressions for $V_\tau(L)$ and $V_\sigma(L)$ which appear in $S_R^{(matt)}$, can be easily obtained, in contrast to the EPRL/FK model, where the expression for $V_\sigma(j)$ can be only defined in the limit of large spins j.

As far as the boundary states are concerned, one will have the spin-foam wavefunctions (instead of spin-network wavefunctions in the spin-foam case) on $\partial M = \Sigma$ with the labels

$$(L_\varepsilon, j_\Delta, \iota_\tau)_{T(\Sigma)} = (L_f, j_l, \iota_v)_{T^*(\Sigma)}\,.$$

This is simply the fact that the boundary of a colored 3-complex is a colored 2-complex, i.e. a spin foam.

5. Conclusions

Categorification of LQG.

Find W and W_{GR}.

Canonical quantization.

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