Wess-Zumino model on nonanticommutative superspaces

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Outline

Introduction

1. M. Dimitrijević, V. Radovanović and J. Wess, JHEP 0712, 059 (2007)
2. M. Dimitrijević and V. Radovanović, JHEP 0904, 108 (2009)
3. M. Dimitrijević, B. Nikolić and V. Radovanović, Phys. Rev. D 81, 105020 (2010)
4. M. Dimitrijević, B. Nikolić and V. Radovanović, Phys. Rev. D 83, 065010 (2011)

Introduction

Quantum field theory encounters problems at high energy/small distances.
Some modifications are needed. One possibility is noncommutativity among space time coordinates. It is given by

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}(x) \ .$$

Canonical noncommutativity

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} = \mathrm{const.}$$

Different modes are constructed on canonical NC space time:
ϕ^4,QED, standard model;
renormalizability, unitarity,...

Introduction

We want to combine supersymmetry with noncommutativity.

- N.Seiberg, JHEP 0306010(2003)

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0 \ , \tag{1}$$

$$[x^\mu \stackrel{\star}{,} x^\nu] = [x^\mu \stackrel{\star}{,} \theta] = [x^\mu \stackrel{\star}{,} \bar\theta] = 0 \tag{2}$$

Deformation of Wess-Zumino model, super YM were analyzed. The product of chiral fields is chiral field. Half SUSY is broken; we get $N = 1/2$ SUSY.

Undeformed Wess Zumino model

The undeformed superspace is generated by x, θ and $\bar{\theta}$ coordinates which fulfill

$$\begin{aligned} [x^m, x^n] &= [x^m, \theta^\alpha] = [x^m, \bar{\theta}_{\dot{\alpha}}] = 0, \\ \{\theta^\alpha, \theta^\beta\} &= \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0, \end{aligned} \tag{3}$$

Every function of the supercoordinates can be expanded in power series in θ and $\bar{\theta}$. For a general superfield $F(x, \theta, \bar{\theta})$ the expansion in θ and $\bar{\theta}$ reads

$$\begin{aligned} F(x, \theta, \bar{\theta}) &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma^m\bar{\theta}v_m \\ &\quad + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\varphi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x). \end{aligned} \tag{4}$$

Undeformed Wess Zumino model

Under the infinitesimal SUSY transformations a general superfield transforms as

$$\delta_\xi F = \left(\xi Q + \bar{\xi}\bar{Q}\right) F, \tag{5}$$

where ξ and $\bar{\xi}$ are constant anticommuting parameters and Q and $\bar{Q}$ are SUSY generators

$$Q_\alpha = \partial_\alpha - i\sigma^m{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m, \tag{6}$$

$$\bar{Q}^{\dot{\alpha}} = \bar{\partial}^{\dot{\alpha}} - i\theta^\alpha\sigma^m{}_{\alpha\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\alpha}}\partial_m. \tag{7}$$

Undeformed Wess Zumino model

The transformation law of the product is given by

$$\begin{aligned} \delta_\xi (F\cdot G) &= \big(\xi Q + \bar{\xi}\bar{Q}\big)(F\cdot G), \\ &= (\delta_\xi F)\cdot G + F\cdot(\delta_\xi G). \end{aligned} \tag{8}$$

The first line tells us that the product of two superfields is a superfield again. The second line is the usual Leibniz rule.

Undeformed Wess Zumino model

A chiral field Φ fulfills $\bar{D}_{\dot{\alpha}}\Phi = 0$, where $\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^{\alpha}\sigma^{m}{}_{\alpha\dot{\alpha}}\partial_m$ is the supercovariant derivative. In terms of component fields the chiral superfield Φ is given by

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= A(x) + \sqrt{2}\theta^{\alpha}\psi_{\alpha}(x) + \theta\theta H(x) + i\theta\sigma^{l}\bar{\theta}(\partial_l A(x)) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^{\alpha}(x))\sigma^{m}{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\Box A(x)). \quad (9)\end{aligned}$$

Undeformed Wess Zumino model

In the undeformed theory, Wess-Zumino Lagrangian is given by

$$\mathcal{L} = \Phi^{+} \cdot \Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \Big(\frac{m}{2}\Phi \cdot \Phi\Big|_{\theta\theta} + \frac{\lambda}{3}\Phi \cdot \Phi \cdot \Phi\Big|_{\theta\theta} + \text{ c.c. } \Big), \quad (10)$$

where m and λ are real constants.

$$\begin{aligned} \mathcal{L} &= A^*\Box A + i(\partial_m\bar{\psi})\bar{\sigma}^m\psi + H^*H \\ &+\frac{m}{2}\Big(2AH - \psi\psi + 2A^*H^* - \bar{\psi}\bar{\psi}\Big) \\ &+\lambda\Big(HA^2 - A\psi\psi + H^*(A^*)^2 - A^*\bar{\psi}\bar{\psi}\Big) \end{aligned} \quad (11)$$

D-deformed WZ model

We introduce a deformation of the Hopf algebra of infinitesimal SUSY transformations by choosing the twist $\mathcal{F}$ in the following way

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}, \tag{12}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$. The inverse of the twist (12),

$$\mathcal{F}^{-1} = e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}, \tag{13}$$

defines the $\star$-product.

D-deformed WZ model

For arbitrary superfields F and G the $\star$-product reads

$$\begin{aligned} F \star G &= \mu_\star\{F \otimes G\} \\ &= \mu\{\mathcal{F}^{-1} F \otimes G\} \\ &= F \cdot G - \frac{1}{2}(-1)^{|F|} C^{\alpha\beta}(D_\alpha F) \cdot (D_\beta G) \\ &\quad - \frac{1}{8} C^{\alpha\beta} C^{\gamma\delta}(D_\alpha D_\gamma F) \cdot (D_\beta D_\delta G), \end{aligned} \tag{14}$$

where $|F| = 1$ if F is odd (fermionic) and $|F| = 0$ if F is even (bosonic).

D-deformed WZ model

The $\star$-product is associative, not Hermitian,
The $\star$-product leads to

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0,$$
$$[x^m \stackrel{\star}{,} x^n] = -C^{\alpha\beta}(\sigma^{mn}\varepsilon)_{\alpha\beta}\bar\theta\bar\theta,$$
$$[x^m \stackrel{\star}{,} \theta^\alpha] = -iC^{\alpha\beta}\sigma^m_{\beta\dot\beta}\bar\theta^{\dot\beta}, \quad [x^m \stackrel{\star}{,} \bar\theta_{\dot\alpha}] = 0. \qquad (15)$$

D-deformed WZ model

The deformed infinitesimal SUSY transformation is defined in the following way

$$\delta^\star_\xi F = \big(\xi Q + \bar\xi\bar Q\big) F. \tag{16}$$

Since the coproduct is not deformed, the Leibniz rule is undeformed. The $\star$-product of two superfields is again a superfield; its transformation law is given by

$$\begin{aligned}\delta^\star_\xi (F\star G) &= \big(\xi Q + \bar\xi\bar Q\big)(F\star G)\\ &= (\delta^\star_\xi F)\star G + F\star(\delta^\star_\xi G).\end{aligned} \tag{17}$$

D-deformed WZ model

The $\star$-product of two chiral fields reads

$$\Phi \star \Phi = \Phi \cdot \Phi - \frac{1}{8} C^{\alpha\beta} C^{\gamma\delta} D_\alpha D_\gamma \Phi D_\beta D_\delta \Phi \tag{18}$$

The (anti)chiral and transversal irreducible components of $\Phi \star \Phi$ are obtained by (anti)chiral and transversal projectors

$$P_1 = \frac{1}{16} \frac{D^2 \bar{D}^2}{\Box}, \tag{19}$$

$$P_2 = \frac{1}{16} \frac{\bar{D}^2 D^2}{\Box}, \tag{20}$$

$$P_T = -\frac{1}{8} \frac{D \bar{D}^2 D}{\Box}. \tag{21}$$

D-deformed WZ model

Thus, we propose the following action

$$\begin{aligned} S &= \int \mathrm{d}^4x \, \Big\{ \Phi^+ \star \Phi \Big|_{\theta\theta\bar\theta\bar\theta} + \Big[\frac{m}{2} \Big(P_2(\Phi\star\Phi)\Big|_{\theta\theta} + 2a_1 P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta} \Big) \\ &+ \frac{\lambda}{3}\Big(P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} + 3a_2 P_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} \\ &+ 2a_3(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta} + 3a_4 P_1(\Phi\star\Phi)\star\Phi^+\Big|_{\bar\theta\bar\theta} \\ &+ 3a_5 \bar{C}^2 P_2(\Phi\star\Phi)\star\Phi^+\Big|_{\theta\theta\bar\theta\bar\theta}\Big) + \text{ c.c. } \Big]\Big\}. \end{aligned} \tag{22}$$

Coefficients $a_1, \ldots, a_5$ are real and constant.

D-deformed WZ model: renormalizability

We calculate the one-loop divergent part of the effective action up to second order in the deformation parameter. We use the background field method, dimensional regularization and the supergraph technique. Note that the use of the supergraph technique significantly simplifies calculations. The kinetic and interaction parts take the form

$$S_0 = \int \mathrm{d}^8 z \left\{ \Phi^+\Phi + \left[-\frac{m}{8}\Phi\frac{D^2}{\Box}\Phi + \frac{ma_1 C^2}{8}(D^2\Phi)\Phi + c.c. \right] \right\}, \quad (23)$$

$$\begin{aligned} S_{int} &= \lambda \int \mathrm{d}^8 z \left\{ -\Phi^2 \frac{D^2}{12\Box}\Phi + \frac{a_2 C^2}{8}\Phi\Phi(D^2\Phi) \right. \\ &\quad - \frac{a_3 C^2}{48}(D^2\Phi)(D^2\Phi)\Phi + \frac{a_4 C^2}{8}\Phi(D^2\Phi)\Phi^+ \\ &\quad \left. + a_5 \bar{C}^2 \Phi\Phi\Phi^+ + c.c. \right\}, \end{aligned} \quad (24)$$

D-deformed WZ model: renormalizability

Following the idea of the background field method, we split the chiral and antichiral superfields into their classical and quantum parts

$$\Phi \to \Phi + \Phi_q, \quad \Phi^+ \to \Phi^+ + \Phi_q^+ \tag{25}$$

and integrate over the quantum superfields in the path integral. Since Φ_q and Φ_q^+ are chiral and antichiral fields, they are constrained by

$$\bar{D}_{\dot\alpha}\Phi_q = D_\alpha \Phi_q^+ = 0.$$

One can introduce unconstrained superfields Σ and Σ^+ such that

$$\begin{aligned} \Phi_q &= -\frac{1}{4}\bar{D}^2\Sigma \ , \\ \Phi_q^+ &= -\frac{1}{4}D^2\Sigma^+ \ . \end{aligned} \tag{26}$$

D-deformed WZ model: renormalizability

From (26) we see that the unconstrained superfields are determined up to a gauge transformation

$$\begin{aligned} \Sigma &\rightarrow \Sigma + \bar{D}_{\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}} \ , \\ \Sigma^{+} &\rightarrow \Sigma^{+} + D^{\alpha}\Lambda_{\alpha} \ , \end{aligned} \tag{27}$$

with the gauge parameter Λ.

D-deformed WZ model: renormalizability

The quadratic part of the classical gauge fixed action in quantum superfields is

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}^8 z \begin{pmatrix} \Sigma & \Sigma^+ \end{pmatrix} \Big(\mathcal{M} + \mathcal{V}\Big) \begin{pmatrix} \Sigma \\ \Sigma^+ \end{pmatrix} \qquad (28)$$

Matrix $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} -m\Box^{1/2}(1 - a_1 C^2\Box)P_- & \Box(P_2 + \xi(P_1 + P_T)) \\ \Box(P_1 + \xi(P_2 + P_T)) & -m\Box^{1/2}(1 - a_1 \bar{C}^2\Box)P_+ \end{pmatrix}. \qquad (29)$$

Matrix $\mathcal{V}$ has the form

$$\mathcal{V} = \begin{pmatrix} F & G \\ \bar{G} & \bar{F} \end{pmatrix} , \qquad (30)$$

D-deformed WZ model: renormalizability

The one-loop effective action is

$$\Gamma = S_0 + S_{int} + \frac{i}{2}\mathrm{Tr}\,\log(1 + \mathcal{M}^{-1}\mathcal{V}) \ . \tag{31}$$

Expansion of the logarithm in (31) gives the one-loop correction to the effective action

$$\Gamma_1 = \frac{i}{2}\mathrm{Tr}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(\mathcal{M}^{-1}\mathcal{V})^n = \sum_{n=1}^{\infty}\Gamma_1^{(n)}. \tag{32}$$

D-deformed WZ model: renormalizability

Result:

$$\Gamma_1^{(2)} = \frac{\lambda^2(1-(2a_1+\frac{a_4}{2}+\frac{2a_5}{m})m^2(C^2+\bar{C}^2))}{4\pi^2\varepsilon}\int d^8z\ \Phi^+\Phi + \frac{\lambda^2(m^2a_3-ma_4-a_5)}{16\pi^2\varepsilon}\int d^8z\ \left[C^2\Phi D^2\Phi + c.c.\right] \quad (33)$$

$$\Gamma_1^{(3)} = -\frac{\lambda^3(ma_1+ma_4+2a_5)}{2\pi^2\varepsilon}\int d^8z\ \left[\bar{C}^2\Phi\Phi\Phi^+ + c.c.\right] + \frac{\lambda^3(2ma_3-a_4)}{8\pi^2\varepsilon}\int d^8z\ \left[C^2\Phi\Phi^+D^2\Phi + c.c.\right]. \quad (34)$$

D-deformed WZ model: renormalizability

From (34) we see that the a_3-term generates divergences of the type

$$\int \mathrm{d}^8 z \ \left[C^2\Phi(z)\Phi^+(z)D^2\Phi(z) + c.c.\right]$$

while the a_1-term and the a_4-term generate divergence of the type

$$\int \mathrm{d}^8 z \ \left[\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.\right].$$

$$\Gamma_1^{(4)} = \frac{\lambda^4}{8\pi^2\varepsilon}\int \mathrm{d}^8 z \ \left[\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z)\left(a_3\bar{D}^2\Phi^+(z) - 2a_4\Phi(z)\right) + c.c.\right] \tag{35}$$

This term does not appear in the classical action. Now we see that if $a_3 = a_4 = 0$ all divergences are the same form as in classical action (necessary condition). We have to absorb all divergences by redefinition of superfield and coupling constants.

D-deformed WZ model: renormalizability

The bare action is given by

$$S_B = S_0 + S_{int} - \Gamma_1^{(2)}\Big|_{dp} - \Gamma_1^{(3)}\Big|_{dp} . \qquad (36)$$

The two-point Green function in (36) gives the renormalization of the superfield Φ

$$\Phi_0 = \sqrt{Z}\Phi , \qquad (37)$$

where

$$Z = 1 - \frac{\lambda^2}{4\pi^2\epsilon}\Big[1 - 2m(C^2 + \bar{C}^2)(ma_1 + a_5)\Big] \qquad (38)$$

and

$$m = Zm_0 \qquad (39)$$

since $\delta_m = 0$. In addition to the field redefinition we obtain the redefinition of the coupling constant

D-deformed WZ model: renormalizability

$$a_{10}C_0^2 = a_1 C^2\Big[1+\frac{\lambda^2 a_5}{2\pi^2\epsilon a_1 m}\Big] \ . \tag{40}$$

From the three-point Green function in (36) we obtain the following conditions

$$\lambda = Z^{3/2}\lambda_0 \ , \tag{41}$$

$$a_{50}\bar{C}_0^2 = a_5\bar{C}^2\Big[1+\frac{\lambda^2(ma_1+2a_5)}{2\pi^2\epsilon a_5}\Big]. \tag{42}$$

Taking these results into account we see that indeed for $a_3 = a_4 = 0$ our model is renormalizable. The case $a_5 = -\frac{1}{2}ma_1$ is interesting. With this choice the divergent part of the three-point function vanishes. This means that there are no divergent counterterms for the coupling constants, i.e. all redefinitions are expressed in terms of the field strength renormalization

Hermitian deformation of WZ model

We define the twist

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta + \frac{1}{2}\bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\partial}^{\dot{\alpha}}\otimes\bar{\partial}^{\dot{\beta}}}. \tag{44}$$

This twist leads to a deformed Leibniz rule. This ensures that the $\star$-product of two superfields is again a superfield. Its transformation law is given by

$$\begin{aligned}
\delta^\star_\xi(\mathrm{F}\star\mathrm{G}) &= \big(\xi Q + \bar{\xi}\bar{Q}\big)(\mathrm{F}\star\mathrm{G}),\\
&= (\delta^\star_\xi\mathrm{F})\star\mathrm{G} + \mathrm{F}\star(\delta^\star_\xi\mathrm{G})\\
&+ \frac{i}{2}C^{\alpha\beta}\Big(\bar{\xi}^{\dot{\gamma}}\sigma^m{}_{\alpha\dot{\gamma}}(\partial_m\mathrm{F})\star(\partial_\beta\mathrm{G}) + (\partial_\alpha\mathrm{F})\star\bar{\xi}^{\dot{\gamma}}\sigma^m{}_{\beta\dot{\gamma}}(\partial_m\mathrm{G})\Big)\\
&- \frac{i}{2}\bar{C}_{\dot{\alpha}\dot{\beta}}\Big(\xi^\alpha\sigma^m{}_{\alpha\dot{\gamma}}\varepsilon^{\dot{\gamma}\dot{\alpha}}(\partial_m\mathrm{F})\star(\bar{\partial}^{\dot{\beta}}\mathrm{G}) + (\bar{\partial}^{\dot{\alpha}}\mathrm{F})\star\xi^\alpha\sigma^m{}_{\alpha\dot{\gamma}}\varepsilon^{\dot{\gamma}\dot{\beta}}(\partial_m
\end{aligned}$$

Hermitian deformation of WZ model

This twist gives a new product in the algebra of superfields called the $\star$-product. For two arbitrary superfields F and G the $\star$-product is defined as follows

$$\begin{aligned} \mathrm{F} \star \mathrm{G} &= \mu_\star\{\mathrm{F} \otimes \mathrm{G}\} \\ &= \mu\{\mathcal{F}^{-1}\,\mathrm{F} \otimes \mathrm{G}\} \\ &= \mu\{e^{-\frac{1}{2}C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta-\frac{1}{2}\bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\partial}^{\dot{\alpha}}\otimes\bar{\partial}^{\dot{\beta}}}\,\mathrm{F} \otimes \mathrm{G}\} \qquad (47) \end{aligned}$$

This $\star$ product is hermitian, associative and nonanticommutative.

Hermitian deformation of WZ model

The $\star$-product leads to

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar{\theta}_{\dot{\alpha}} \stackrel{\star}{,} \bar{\theta}_{\dot{\beta}}\} = \bar{C}_{\dot{\alpha}\dot{\beta}}, \quad \{\theta^\alpha \stackrel{\star}{,} \bar{\theta}_{\dot{\alpha}}\} = 0,$$
$$[x^m \stackrel{\star}{,} x^n] = 0,$$
$$[x^m \stackrel{\star}{,} \theta^\alpha] = 0, \quad [x^m \stackrel{\star}{,} \bar{\theta}_{\dot{\alpha}}] = 0. \tag{48}$$

Hermitian deformation of WZ model

Deformed WZ action

$$S = \int d^4x \left\{ \Phi^+ \star \Phi \Big|_{\theta\theta\bar\theta\bar\theta} + \left[\frac{m}{2} P_2 \big(\Phi \star \Phi \big) \Big|_{\theta\theta} + \frac{\lambda}{3} P_2 \Big(\Phi \star P_2 \big(\Phi \star \Phi \big) \Big) \Big|_{\theta\theta} + \text{ c.c } \right] \right\}. \quad (49)$$

Hermitian deformation of WZ model

The divergent part of the two-point function

$$
\begin{aligned}
\Gamma_1^{(2)} \;=\; & -\frac{m^2\lambda^2 K^m_{\ a} K^{*na}}{24\pi^2\varepsilon}\int \mathrm{d}^4x\,(\partial_m A\partial_n A + \partial_m A^*\partial_n A^*) \\
& -\frac{m^2\lambda^2 K^{mn}K_{mn}}{16\pi^2\varepsilon}\int \mathrm{d}^4x\, F^2 + cc + \frac{\lambda^2}{4\pi^2\varepsilon}\int \mathrm{d}^8z\,\Phi^+\Phi \\
& -\frac{\lambda^2 K^m_{\ a}K^{*na}}{6\pi^2\varepsilon}\int \mathrm{d}^4x\, A^*\left(m^2+\frac{\Box}{6}\right)\partial_m\partial_n A \\
& +\frac{i\lambda^2 K^m_{\ a}K^{*n}_{\ \ b}}{24\pi^2\varepsilon}\int \mathrm{d}^4x\,\bar\psi(\bar\sigma^b\partial^a - \bar\sigma^a\partial^b + i\varepsilon^{abcd}\bar\sigma_d\partial_c)\partial_m\partial_n\psi \\
& -\frac{i\lambda^2 K^m_{\ a}K^{*na}}{12\pi^2\varepsilon}\int \mathrm{d}^4x\,\bar\psi\bar\sigma_n\partial_m\left(m^2-\frac{\Box}{6}\right)\psi \\
& +\frac{i\lambda^2 K^m_{\ a}K^{*na}}{72\pi^2\varepsilon}\int \mathrm{d}^4x\,\bar\psi\bar\sigma^l\partial_l\partial_m\partial_n\psi
\end{aligned}
$$

Hermitian deformation of WZ model

$$+\frac{\lambda^2 K^m_{\ a} K^{*na}}{72\pi^2\varepsilon}\int \mathrm{d}^4x\; F^*\partial_m\partial_n F$$
$$+\frac{m^2\lambda^2 K^m_{\ a} K^{*na}}{12\pi^2\varepsilon}\int \mathrm{d}^4x \mathrm{d}^4y\; \partial_m\partial_n F(x)\Box_x^{-1}\delta(x-y)F^*(y) \quad (50)$$

Most of divergent terms do not have its counterparts in the classical action. Two-point divergences cannot be absorbed by counterterms.

Conclusion

We consider two different deformations:

1. D-deformation

associative, non-Hermitian

Deformed WZ model is renormalizable at one loop and second order in deformation parameter.

2. Hermitian deformation

associative, Hermitian

Deformed WZ model is NOT renormalizable.